ARITHMETISCHE GEOMETRIE OBERSEMINAR

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ÉTALE COHOMOLOGY OF DIAMONDS

At the Berkeley course [5], Scholze outlined a strategy for attaching a (semisimple) L-parameter to an irreducible smooth representation of a p-adic group. The strategy is based on adapting V. Lafforgue's work in the case of global function fields. This requires analogues of moduli spaces of shtukas in the p-adic case, which are spaces that do not belong to classical algebraic geometry. More precisely, the course [5] defined a new category of geometric objects called *diamonds*, and showed that the relevant analogues of moduli spaces of shtukas are diamonds. For the applications, it is necessary to have a solid foundation on the étale cohomology of these objects, and this is the topic of this ARGOS. The goal is to prove the analogues of proper and smooth base change as well as Poincaré duality in this setup; this will allow us to define the "6 functors" Rf_*, f^*, $Rf_!$, $Rf^!$, Hom, $\otimes$.

TALKS

1. Talk: Perfectoid spaces

Define perfectoid rings following [5, Definition 6.1.1.] and the tilting equivalence for such [5, Theorem 6.2.6.]. Introduce the adic space of a perfectoid affinoid ring and state [5, Theorem 6.1.11.], [5, Theorem 7.1.1.]. Define perfectoid spaces [5, Definition 7.1.2.] and their étale site [5, Section 7.5]. Recall the tilting equivalence for perfectoid spaces and their étale sites [5, Theorem 7.4.5.] resp. [5, Corollary 7.5.3.]. Finally give several examples of perfectoid spaces including the perfectoid closed unit disk $\mathrm{Spa}(K\langle T_1^{1/p^\infty}, \cdots, T_n^{1/p^\infty}\rangle)$ and perfections of rigid-analytic varieties in characteristic p, cf. [4, Definition 6.9.].

2. Talk: w-local spaces

Define w-local spectral spaces [1, Definition 2.1.1] and prove [1, Lemma 2.1.4]. Present [1, Lemma 2.1.10] associating to a spectral space X a canonical pro-open cover X^Z by a w-local spectral space. Explain how to endow X^Z with a structure of a scheme if $X = \mathrm{Spec}(A)$ is an affine scheme [1, Remark 2.1.11]. It might be helpful to consult [6, Tag 0975] (and the material preceding the lemma). We will need a similar construction for adic spaces. Follow [5, Section 15.3.] and introduce the perfectoid space X^T for a qcqs perfectoid space X, and follow the section on w-local spaces in [3].

3. Talk: Topologies on (Perf)

Introduce pro-étale morphisms of perfectoid spaces [5, Definition 8.2.1.]. Define the category (Perf) and the pro-étale topology on it [5, Definition 8.2.5]. Prove [5, Proposition 8.2.7]. Present [5, Section 9.1., Section 9.2.]. Define the faithful (also called v-) topology on (Perf) [5, Definition 15.3.3.] and prove [5, Theorem 15.3.5.]. Mention its corollaries [5, Corollary 15.3.7.] and [5, Corollary 15.3.8.].

4. Talk: Descent

Discuss the notions of closed immersions and separated maps of adic spaces. Then, following the section on descent in [3], prove various effective descent statements in the

v-topology: Affinoid perfectoid spaces over w-local spaces, pro-étale separated maps over w-local spaces, étale separated maps over general spaces.

5. Talk: Diamonds

Define the category of diamonds, and prove various basic facts, following the section on diamonds in [3]. In particular, show that diamonds are v-sheaves. Moreover, define the underlying topological space of a diamond, and introduce spatial diamonds. Finally, give the criterion for when a "v-diamond" is a diamond.

6. Talk: The étale site of spatial diamonds

Introduce the small v-, pro-étale, and étale site of a diamond, following the corresponding section in [3]. Show that for diamonds arising from rigid spaces, this recovers the étale site of the rigid space. Show that for a spatial diamond, the pullback functors from the étale to the pro-étale and v-topos are fully faithful and leave cohomology invariant. Moreover, show that étale cohomology takes (nice) inverse limits of diamonds to direct limits.

7. Talk: Proper base change

Introduce the notion of proper maps of (spatial) diamonds, as partially proper and quasicompact maps. Show that the proper base change theorem holds true in this setup.

Moreover, discuss the functorial compactification of taut morphisms, and use this to define the $Rf_!$ functor, and extend the proper base change theorem.

8. Talk: Smooth morphisms

After briefly recalling the notion of smooth morphisms in rigid geometry, define smooth morphisms of (spatial) diamonds. Show that with the usual definition of constructible sheaves, $R^i f_!$ preserves this class if f is smooth, separated and quasicompact.

9. Talk: The trace map and Deligne's fundamental lemma

Define the trace map $R^{2d} f_! f^* \mathcal{F} \to \mathcal{F}(-d)$ for f smooth of dimension d, separated and quasicompact. Prove a version of Deligne's fundamental lemma, cf. [2, Theorem 7.4.2].

10. Talk: Smooth base change and Poincaré duality

Prove the smooth base change theorem, cf. [2, Theorem 4.5.1] in the case of adic spaces. Moreover, introduce the functor $Rf^!$ and identify it in the case of a smooth morphism. In particular, deduce Poincaré duality.

References

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