The Relation of Cobordism to K-Theories

PhD student seminar winter term 2006/07

November 7, 2006

In the current winter term 2006/07 we want to learn something about the different flavours of cobordism theory - about its geometric constructions and highly algebraic calculations using spectral sequences. Further we want to learn the modern language of Thom spectra and survey some levels in the tower of the following Thom spectra

$$MO \leftarrow MSO \leftarrow MSpin \leftarrow \ldots \leftarrow M\{e\}.$$

After that there should be time for various special topics: we could calculate the homology or homotopy of Thom spectra, have a look at the theorem of Hartori-Stong, care about d-,e- and f-invariants, look at K-, KO- and MU-orientations and last but not least prove some theorems of Conner-Floyd type:

$$MU_*X \otimes_{MU_*} K_* \cong K_*X.$$

Unfortunately, there is not a single book that covers all of this stuff or at least a main part of it. But on the other hand it is not a big problem to use several books at a time. As a motivating introduction I highly recommend the slides of Neil Strickland. The 9 slides contain a lot of pictures, it is fun reading them!

- November 2nd, 2006:
  Talk 1 by Marc Siegmund: This should be an introduction to bordism, tell us the geometric constructions and mention the ring structure of Ω_*^G. You might take the books of Switzer (chapter 12) and Bröker/tom Dieck (chapter 2).
  Talk 2 by Julia Singer: The Pontrjagin-Thom construction should be given in detail. Have a look at the books of Hirsch, Switzer (chapter 12) or Bröker/tom Dieck (chapter 3). But Hirsch is perhaps the best choice.

- November 9th, 2006:
  Talk 3 by Saeid Hamzeh Zarghani: On classifying spaces: Give the definition of a classifying space BG and explain the construction by Graeme Segal as the realization of the nerve of a group considered as a category. Then take the orthogonal group O and give the construction of BO by Grassmannian manifolds. At the end compute the cohomology of BO as in chapter 16 of the book of Switzer. Probably there is also time to explain other examples like $B\{e\}$, $B\mathbb{Z}/2$, $B\mathbb{Z}$, BS^1 and BU.
  Talk 4 by Juan Wang: Define Thom spectra (as in Adams74 p. 135). Then explain

what an X-structure (as in Switzer chapter 12) or a π-structure (as in Miller p. 39),resp.,is. The tower of classifying spaces (Miller p.40) and the thom tower then is just a gimmick, so mention it. Give some further examples of Thom spectra.

- November 21st, 2006:
  Talk 5 by Maria Castillo: At this point we can give some applications since we have worked on cobordism theory and characteristic classes (as polynomial generators of classifying spaces). Also it is a good repetition of the stuff of the seminar in the summer semester. So we should talk about the $\hat{A}$-genus and the Todd-genus. Have a look at the book of Hirzebruch, Berger, Jung (section 1.7 + 1.8).
  Talk 6 by Hanno von Bodecker: Examples

- December 7th, 2006:
  Talk 7 by Martin Langer: Calculations part I: choose your favourite group G and compute H_*MG, K_*MG, π_*MG or MG_*MG (perhaps for $G = O$).
  Talk 8 by Johannes Kuhr: Calculations part II: alternative calculations, Hartori-Stong.

- January 11th, 2007:
  Talk 9 by Balazs Visy: Introduction to the d-,e- and f-invariants as in the paper of Adams68 and Laures.
  Talk 10 by Hanno von Bodecker: Continuation and further aspects of the d-, e- and f-invariants.

- January 25th, 2007:
  Talk 11 by Rui Wang: Special topics I: One option is to consider K-, KO- and MU-orientations as in the book of Rudyak.
  Talk 12 by Ferit Deniz: Special topics II.

- February 8th, 2007:
  Talk 13 by Holger Reeker: Special Topics III: Theorems of Conner-Floyd type. One option is to take the book of Conner-Floyd and the papers of Hovey and Hopkins.
  Talk 14 by Arne Weiner: Special Topics IV.

Our PhD-seminar takes place 15:00 - 18:00 in room NA 1/64. A talk should take 75 minutes. If you have any questions, please send an email to `holger.reeker@ruhr-uni-bochum.de`.

References

[Ada68] J.F. Adams: On the Groups $J(X)$-IV.

[Ada74] J.F. Adams: Stable Homotopy and Generalized Homology.

[BtD] Bröker, tom Dieck: Kobordismentheorie, LNM 178.

[CF] Conner, Floyd: The Relation of Cobordism to K-Theories, LNM 28.

[Hi] Hirsch: Differential Topology, GTM 33.

[HBJ] Hirzebruch, Berger, Jung: Manifolds and Modular Forms.

[HoHo] Hopkins, Hovey: Spin Cobordism determines Real K-Theory.

[Ho] Hovey: Spin Bordism and Elliptic Homology.

[Koch] Kochman: Bordism, Stable Homotopy and Adams Spectral Sequences.

[La] Laures: The topological q-expansion principle
`www.ruhr-uni-bochum.de/topologie`

[Mi] Miller: Notes on Cobordism: `www.ruhr-uni-bochum.de/topologie`

[Rud] Rudyak: On Thom Spectra, Orientability, and Cobordism.

[Sato] Sato: Algebraic Topology: An Intuitive Approach. Transl. of Math. Mono., AMS.

[Seg] Segal: Classifying Spaces and Spectral Sequences.

[Sto] Stong: Notes on Cobordism Theory.

[Str] Strickland: Cobordism and formal power series:
`http://neil-strickland.staff.shef.ac.uk/durham.pdf`

[Swi] Switzer: Algebraic Topology - Homology and Homotopy