Problem sheet 4 Rigid analytic geometry Winter term 2024/25

Problem 1 (5 points). *Let X be a G_+-space which is locally G_* and quasi-separated. Show that X satisfies condition B from Definition 1.2.16: If $N \in \mathbb{N}$ and $U = \bigcup_{i=1}^{N} \Omega_i$ where $\Omega_i \in \mathfrak{Qc}(X)$ then $[\Omega_1, \ldots, \Omega_N] \models U$.*

Problem 2 (11 points). *Let F be an ordered field. Let $\mathfrak{B}$ be the set of all intervals $(a, b)_F$ with $a \leq b \in F \cup \{\pm\infty\}$. We equip F with the topology for which $\mathfrak{B}$ is a topology base and the G_+-topology obtained by forcing the elements of $\mathfrak{B}$ to be quasicompact. Describe a bijection between the set F^* of van der Put points of F and an appropriately defined set of Dedekind cuts.*

Problem 3 (5 points). *Let X be a topological space which is compact (and thus Hausdorff). Show that the following conditions are equivalent:*

- *X is a spectral space.*
- *The clopen subsets of X form a topology base for X.*
- *X is totally disconnected in the sense that no connected subset of X has more than one element.*
- *The map $X \to \pi_0(X)$ is a homeomorphism.*

Problem 4 (1 point). *When X satisfies the conditions of the previous problems and Y is an arbitrary spectral space, show that a map $Y \to X$ is spectral if and only if it is continuous.*

Two of the 22 points from this sheet are bonus points which are not counted in the calulation of the 50%-threshold for passing the exams.

Solutions should be e-mailed to my institute e-mail address (my second name (franke) at math dot uni hyphen bonn dot de) before Monday November 18.