



Minisymposium 24 - Probability and Geometry

Stochastic Processes as Rough Paths and Carnot-Caratheodory geometry

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Brownian motion on the step- n free nilpotent group with d generators is a well-known object; in particular, there are Gaussian heat-kernel bounds in term of the Carnot-Caratheodory metric. The resulting sample path regularity is exactly the required regularity in the sense of Lyons' rough path theory. In fact, it suffices to consider $n=2$, that is, standard Brownian motion and Levy's area. If one replaces standard Brownian motion by (i) a continuous martingale or (ii) a suitable Gaussian process there are alternative ways to construct Levy's area. The required sample path regularity in the rough path sense can be shown via old ideas from Lepingle and Wiener-Ito chaos integrability respectively.

Finally, if one considers (iii) Markov process with uniformly sub-elliptic generators the theory of Dirichlet forms yields Gaussian heat-kernel bounds and we obtain a large class of rough paths. A support description on path space was conjectured by T. Lyons and we will report on some progress in this direction. Joint work with N. Victoir.