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A stochastic Datko-Pazy theorem

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The well-known Datko-Pazy theorem states that if $(T(t))_{t\geq 0}$ is a strongly continuous semigroup on a Banach space E such that all orbits $T(\cdot)x$ belong to the space $L^p(\mathbb{R}_+, E)$ for some $p \in [1, \infty)$, then $(T(t))_{t\geq 0}$ is uniformly exponentially stable, or equivalently, there exists an $\epsilon > 0$ such that all orbits $t \mapsto e^{\epsilon t}T(t)x$ belong to $L^p(\mathbb{R}_+, E)$. We show that a similar result also hold for so-called γ–radonifying operators, namely the equivalence of

1. For all $x \in E$, $T(\cdot)x \in \gamma(\mathbb{R}_+, E)$.
2. There exists an $\epsilon > 0$ such that for all $x \in E$, $t \mapsto e^{\epsilon t}T(t)x \in \gamma(\mathbb{R}_+, E)$.

If E is a Hilbert space, $\gamma(\mathbb{R}_+, E) = L^2(\mathbb{R}_+, E)$ and we reobtain Datko's theorem mentioned above. γ–radonifying operators play an important role in the study abstract stochastic Cauchy problems on E whence the result can also be seen as a perturbation result for stochastic Cauchy problems.

[1] B. Haak, M. Veraar, J. van Neerven: *A stochastic Datko-Pazy theorem*, submitted; avaiable on ArXiv.