Minisymposium 6 - Positive definite functions and applications

Transformations of moment sequences, a fix-point-measure and its Mellin transform

CHRISTIAN BERG (UNIVERSITY OF COPENHAGEN)

In recent papers A. Durán and the speaker studied some non-linear transformations from Hausdorff moment sequences (a_n) to Stieltjes moment sequences (s_n), namely

$$s_n = (a_0 a_1 \cdots a_n)^{-1}, \quad s_n = (a_0 + a_1 + \cdots + a_n)^{-1}.$$

This made it possible to unify different constructions from the theory of additive functionals of Lévy-processes. The 'sum' transformation has a fix-point (m_n) defined by the recursive equation

$$(m_0 + m_1 + \cdots + m_n)m_n = 1, \quad n \geq 0$$

i.e.

$$m_0 = 1, \quad m_1 = \frac{-1+\sqrt{5}}{2}, \quad m_2 = \frac{\sqrt{22+2\sqrt{5}}-\sqrt{5}-1}{4}, \ldots,$$

and (m_n) is the moment sequence of a probability measure μ on $[0,1]$. In a new manuscript we prove that μ has an increasing and convex density and that the Mellin transform F of μ

$$F(z) = \int_0^1 t^z \, d\mu(t),$$

can be characterized in analogy with the Bohr-Mollerup theorem about the Gamma function as the unique log-convex function $F :]-1, \infty[\to]0, \infty[$ satisfying $F(0) = 1$ and the functional equation

$$1/F(s) = 1/F(s+1) - F(s+1), \quad s > -1.$$

We also prove that F extends to a meromorphic function in the whole complex plane.